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A MODEL OF NANOFLUIDS EFFECTIVE THERMAL CONDUCTIVITY BASED ON DIMENSIONLESS GROUPS

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Thermal conductivity is an important parameter in the field of nanofluid heat transfer. This article presents a novel model for the prediction of the effective thermal conductivity of nanofluids based on dimensionless groups. The model expresses the thermal conductivity of a nanofluid as a function of the thermal conductivity of the solid and liquid, their volume fractions, particle size and interfacial shell properties. According to this model, thermal conductivity changes nonlinearly with nanoparticle loading. The results are in good agreement with the experimental data of alumina-water and alumina-ethylene glycol based nanofluids.

Keywords: *dimensionless group, model, nanofluid, particle size, thermal conductivity*

Introduction

Nanofluids are prepared by dispersing solid nanoparticles in fluids such as water, oil, or ethylene glycol. These fluids represent an innovative way to increase thermal conductivity and, therefore, heat transfer. Unlike heat transfer in conventional fluids, the exceptionally high thermal conductivity of nanofluids provides for exceptional heat transfer, a unique feature of nanofluids. Advances in device miniaturization have necessitated heat transfer systems that are small in size, lightmass, and high-performance. In many of these novel systems, nanofluids have been developed because of their intriguing properties, such as: high thermal conductivity, stability and the fact that they do not clog micro-channels. Investigations of the enhanced thermal conductivity of nanofluids have only recently been made. For example, anomalously large increases in thermal conductivity have been observed for nanofluids of the following materials: cupric oxide (CuO) [1, 2], copper (Cu) [3, 4], gold (Au) [5], silver (Ag) [5], alumina (Al_2O_3) [1, 2, 6], carbon nanotubes [7, 8] and titania (TiO_2) [9]. From a theoretical point of view, the anomalous thermal conductivity enhancement is interesting because measured thermal conductivities are much larger than those that are predicted from existing models. Conventional models of effective thermal conductivity of suspensions are listed in Table 1, where k_{eff} is the effective thermal conductivity of the suspension, k_m and k_d are the thermal conductivities of the suspending medium and solid particle respectively, n is a shape factor for the particle, ν is the particle volume fraction, $\alpha=(k_d/k_m)$, $\beta=(\alpha-1)/(\alpha+2)$, α and β are empirical fitting parameters. The shape factor, n, is given by:

Table 1 Common models for estimating effective thermal conductivities of suspensions

Model	Expression
Maxwell [10]:	$\frac{k_{eff}}{k_m}=1+\frac{3(\alpha-1)\nu}{(\alpha+2)-(\alpha-1)\nu}$
Hamilton and Crosser (H–C) [11]:	$\frac{k_{eff}}{k_m}=\frac{\alpha+(n-1)-(n-1)(1-\alpha)\nu}{\alpha+(n-1)+(1-\alpha)\nu}$
Jeffery [12]:	$\frac{k_{eff}}{k_m}=1+3\beta\nu+\nu^2\left(3\beta^2+\frac{3\beta^2}{4}+\frac{9\beta^3}{16}\frac{\alpha+2}{2\alpha+3}+\frac{3\beta^4}{20}\ldots\right)$
Davis [13]:	$\frac{k_{eff}}{k_m}=1+\frac{3(\alpha-1)}{(\alpha+2)-(\alpha-1)\nu}[\nu+f(\alpha)\nu^2+O(\nu^3)]$

$$n=3/\lambda$$

where λ is defined as the ratio of the surface area of a sphere with a volume equal to that of the particle to the actual surface area of the particle.

For nanofluids, the thermal conductivities estimated by these models deviate far from experimental data because they neglect the effect of particle size and solid-liquid interfacial properties. The neglected factors have an important influence on the enhancement of the thermal conductivity [14]. Layering of the

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liquid at the solid interface may enhance thermal conductivity, because the atomic structure of the liquid layer is much more ordered than that of bulk liquid [15]. Thus, this layer has higher thermal conductivity than the bulk liquid; this leads to more heat transfer.

Modeling thermal conductivity

Through the use of appropriate dimensionless groups, a new expression to estimate the effective thermal conductivity of nanofluids will be derived in this section. First, the effective thermal conductivity for the nanofluid (k_{eff}) will be considered to be a function of the thermal conductivities of the base fluid (k_m), the solid particle (k_p), the interfacial shell (k_i), the particle diameter (d_p), the volume fraction of the particle (ν_p), and the interfacial shell thickness (t):

$$k_{eff}=f(k_m, k_p, k_i, d_p, \nu_p, t) \quad (1)$$

Dimensionless groups are then defined as follows [16, 17]:

$$\pi_1=k_{eff}/k_m \quad (2)$$

$$\pi_2=k_i/k_p \quad (3)$$

$$\pi_3=t/d_p \quad (4)$$

$$\pi_4=\nu_p \quad (5)$$

Consider one of dimensionless groups to be a function of other groups as follows:

$$\pi_1=k_{eff}/k_m=f_1(k_i/k_p, t/d_p, \nu_p) \quad (6)$$

For nanofluids:

$$k_{eff}>k_m \Rightarrow (k_{eff}/k_m)>1 \Rightarrow (k_{eff}/k_m)=1+R \quad (7)$$

where R is an enhancement factor and is expressed as follows:

$$R=\gamma\left[\left(\frac{k_i}{k_p}\right)^a\left(\frac{t}{d_p}\right)^b(\nu_p)^c\right] \quad (8)$$

By combining Eqs (7) and (8), a general form of the dimensionless equation for the effective thermal conductivity of nanofluids is obtained as:

$$\frac{k_{eff}}{k_m}=1+\gamma\left[\left(\frac{k_i}{k_p}\right)^a\left(\frac{t}{d_p}\right)^b(\nu_p)^c\right] \quad (9)$$

where γ, a, b and c are constants that depend on the type of nanofluid.

For a particular nanofluid system, where k_p, k_i and t are constant, Eq. (9) simplifies into:

$$\frac{k_{eff}}{k_m}=1+m\frac{(\nu_p)^\alpha}{(d_p)^\beta} \quad (10)$$

where m is a factor that depends on the properties of the solid particle and interfacial shell, while α and β are empirical constants determined from experimental data. In this work, experimental data of alumina–water and alumina–ethylene glycol nanofluids were used [18–20]. The thermal conductivities of ethylene glycol (EG) and water were taken to be 0.252 and 0.604 W m^{-1} K^{-1}, respectively. The values of m, α and β were calculated from experimental data [16, 17] using least-squares regression (Table 2).

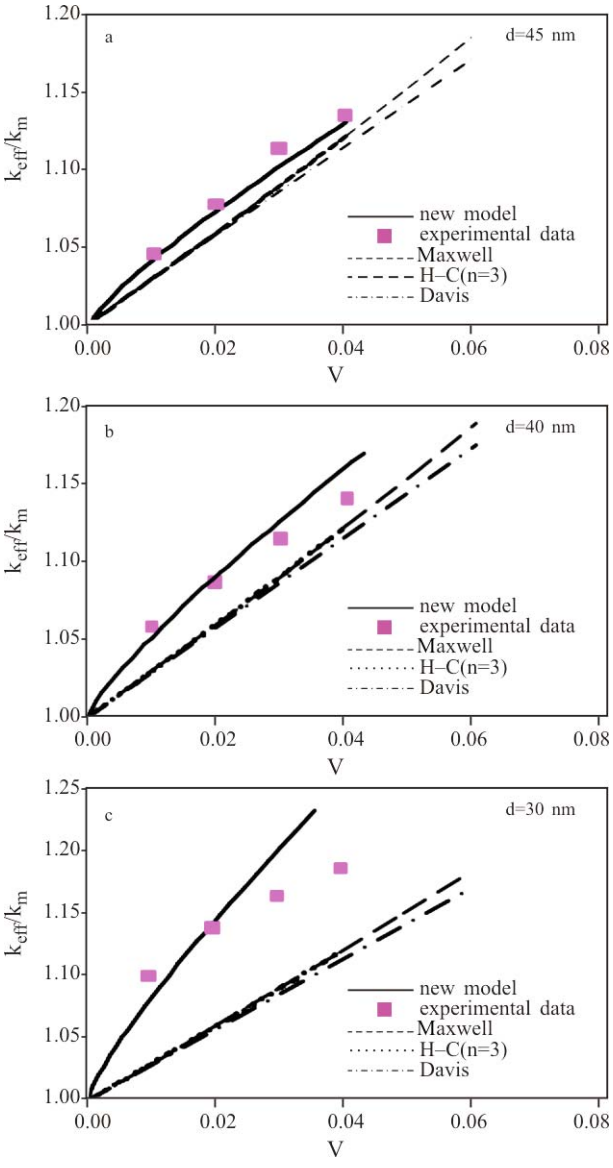

Fig. 1 Comparison of model predictions with experimental data for Al_2O_3–water nanofluids

Table 2 Empirical constants for Al_2O_3–water and Al_2O_3–EG systems

Nanofluids	Constants		
	m	α	*B*
Al_2O_3–water	1128.326	0.825	1.688
Al_2O_3–EG	233.066	1.294	0.95

Results and discussion

Figures 1a–c and 2a, b show comparisons of our new model predictions for Al_2O_3–water and Al_2O_3–EG nanofluids respectively, compared to experimental data [14, 15, 18] and to previous models. It is clear from the figure, that the model (described in Eq. (10)) is in good agreement with experimental data and outperforms previously derived models when applied to nanofluids. Furthermore, the proposed model creates a non-linear relation between the effective thermal conductivity and nanoparticle volume fraction. Previous models are linear and deviate from experimental data of nanofluids. Because previously derived models do not consider the effects of particle size, the deviation in the predicted values of the effective thermal conductivity for small particle sizes becomes significant.

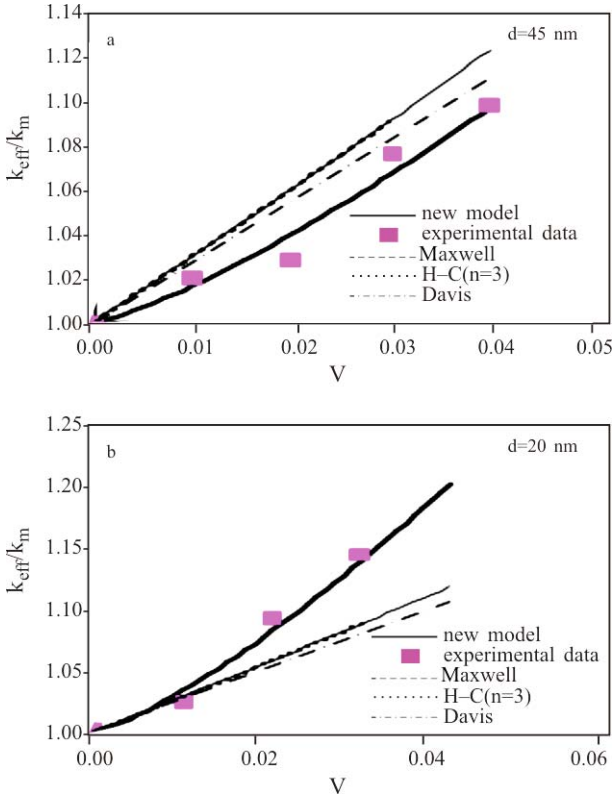

Fig. 2 Comparison of the new model prediction with the experimental data and previous models for the Al_2O_3–EG systems

Also as seen in these figures for Al_2O_3–water and Al_2O_3–EG nanofluids, the modeled effective thermal conductivity increases as particle size is reduced. This phenomenon is due to the relative effects of nanoparticle motion mechanisms of dilute suspensions such as Brownian motion, thermophoresis and osmophoresis, including size dependence, on the thermal conductivity [21]. Also, it may be due to the effect of effective surface increasing with particle size decreasing.

Conclusions

A model for the effective thermal conductivity of nanofluids based on dimensionless groups has been presented. Comparing with conventional models, the new expression not only considers the dependence of thermal conductivity on the solid particle, base fluid and their volume fraction, but also the particle size and the properties of the interfacial shell. This model indicate that increase the thermal conductivity of nanofluid with particle size decreasing. Although new model has a simple form, it is in good agreement with experimental data for Al_2O_3–water and Al_2O_3–EG nanofluids. The model could be readily applied to other nanofluids.

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